

Whole Number Division Algorithm

Question:

How can you get an answer to $123 \div 8$?

Possible answers to the question are:

1. I did it on a calculator. The answer is big. It is .06504065.
2. I did it on a calculator. The answer is 15 remainder 375.
3. I'll show you what I was taught years ago. I didn't (and still don't) understand why the method works but it got the right answer according to the teacher.

$$\begin{array}{r}
 15 \\
 8 \overline{) 123} \\
 \underline{- 8} \\
 43 \\
 \underline{- 40} \\
 3
 \end{array}$$

4. I saw a strange method in my kid's grade 5 class for doing divisions. The numbers were in places I had never seen when I was in school but I was making some sense of why the numbers were there. For example, I think 80 comes from 8×10 and 15 is 10 and 5 added.

$$\begin{array}{r}
 8 \overline{) 123} \\
 \underline{- 80} \quad | \quad 10 \\
 43 \\
 \underline{- 40} \quad | \quad 5 \\
 3 \quad | \quad 15
 \end{array}$$

5. I saw a grade 5 student doing this scribbling. It didn't make much sense to me but the circled answer was correct.

$$\begin{array}{r}
 123 - 32 \\
 91 - 8 \\
 83 - 32 \\
 51 - 8 \\
 43 - 32 \\
 11 - 8 \\
 3
 \end{array}$$

$$\begin{array}{cccccc}
 4 & 1 & 4 & 1 & 4 & 1 \\
 & & \textcircled{15} & & &
 \end{array}$$

Response 1 indicates a lack of empowerment about doing arithmetic and is about as far out in “left field” as you can get. The division result is incorrect. $123 \div 8$ has to be a number larger than 1 because 123 is greater than 8. The responder divided 8 by 123 rather than 123 by 8. Also, .06504065 is not a big number. It is around 65 thousandths (not 65 thousands).

Response 2 indicates a lack of empowerment about doing arithmetic but at least shows proficiency with entering numbers into a calculator namely, entering 123, then \div , then 8. This order is contrary to the “goes into” division algorithm that has been taught for many years (see response 3). The remainder comment is incorrect. The remainder is .375 (not 375). .375 is a decimal way of representing the whole number remainder of 3.

Response 3 concerns the “goes into” algorithm. It is what most adults were taught in elementary school. The method is efficient but does not reveal any understanding about why it works. It was taught because in the “good old days” part of the purpose of elementary school mathematics curricula was to train the mind to be an efficient and error free calculator (electronic calculating devices were not around/very expensive in those days).

Response 4 is about a particular style of writing for the subtractive division algorithm. The algorithm allows for personal comfort with numbers and does not require any division facts knowledge. It relies on multiplication facts knowledge.

Response 5 indicates a general lack of understanding about division on the part of the responder. Although an unorthodox style of writing, the student’s scribbling is a valid way to divide. The student is trying to reach zero by removing groups of 8 and keeps track of how many groups of 8 are removed. The student seems limited by multiplication facts knowledge (knows 1×8 and 4×8) and by an ability to subtract when the ones digit of the number being subtracted from is smaller than the ones digit of the number being subtracted.

The Subtractive Division Algorithm

The “goes into” algorithm that you were likely taught in elementary school is an efficient way of doing division but it obscures what is going on. In other words, it does not support an understanding of why the algorithm works. The subtractive algorithm described here has three advantages: (1) it requires no division facts knowledge, (2) it supports an understanding of why the algorithm works, and (3) it allows for varying student proficiency with multiplication and subtraction.

The subtractive algorithm relies on understanding division as splitting up into equal groups and that the goal is to “share” all of what one starts with (although there can be a remainder).

Consider $696 \div 6$.

A “slow” way to obtain the answer is shown here, using a particular writing style. There are other styles (shown later on).

The thinking involved is to remove groups of 6 until that cannot be done any more. You have to keep track of how many groups of 6 are removed (the numbers on the right side of the vertical line).

The person doing the division has the option to remove large or small amounts of groups of 6. That decision depends on the multiplication power of the individual. In the “goes into” algorithm, there is no choice. You have to find the largest amount possible each time.

$$\begin{array}{r} 6 \overline{) 696} \quad \begin{array}{l} 40 \\ 40 \\ 30 \\ 6 \end{array} \\ \underline{- 240} \\ 456 \\ \underline{- 240} \\ 216 \\ \underline{- 180} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$$

Here is a faster way to obtain the answer. It maximizes the amount removed according to place value positions - the largest number of 100s' is removed, then the largest number of 10s, and finally the largest number of 1s'. This is precisely what the “goes into” algorithm does but you would be hard-pressed to recognize that because of the writing style and the way it was taught to you.

$$\begin{array}{r} 6 \overline{) 696} \quad \begin{array}{l} 100 \\ 10 \\ 6 \end{array} \\ \underline{- 600} \\ 96 \\ \underline{- 60} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$$

Finally, there is a writing style that almost looks like the “goes into” algorithm. It may help you realize what is going on with the “goes into” algorithm. Here it is.

$$\begin{array}{r} \begin{array}{l} 6 \\ 10 \\ 100 \end{array} \overline{) 116} \\ 6 \overline{) 696} \\ \underline{- 600} \\ 96 \\ \underline{- 60} \\ 36 \\ \underline{- 36} \\ 0 \end{array}$$

Note:

One matter must be made clear. When doing the subtractive algorithm, you DO NOT think how much does this go into that. You think what multiple of the divisor can be subtracted from what is there “to share”. That thinking does not have to follow place value positions. Here is an example.

Notice that 1024 is subtracted (32 x 32) rather than 960 (32 x 30). Why? Because I happen to know that 32 x 32 is 1024. So why not subtract a lot from what is there to share rather than a smaller amount based on maximizing a place value position.

$$\begin{array}{r} 2 \\ 32 \overline{) 34} \\ \hline 32 \overline{) 1100} \\ - 1024 \\ \hline 76 \\ - 64 \\ \hline 12 \end{array}$$

Refer to: [Grade 4 Division algorithm](#) if more help is needed.